sum, ergo cogito

about Math and Poetry, no longer as separate disciplines, but rather linked in harmony.
Three Exciting Educational Journals

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Center for
auto SOCRATIC EXCELLENCE

sum, ergo cogito
Dedicated to the Mathematician and Poet residing in all of us.

The following are nine mathematical research articles with some form of poetry included in each. Some are written entirely in rhyme, some finish in rhyme, and some concluding with the Japanese haiku.

What have poetry and math to do with one another? Maybe nothing. Maybe everything.

What I’ve found is for me to write a poem about anything, I have to understand the material very well. Research precedes the poem.

You be the judge.
sum, ergo cogito
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Center for auto SOCRATIC EXCELLENCE
sum, ergo cogito
6
The Quadrilateral Jamboree!

This rhyme attacks all of the formulas that exist to help the student calculate “areas”. How many are really necessary? The hero of the poem, the “meek small figure”, says it all!

The room abuzz, the people talking,
Down the aisle the shapes came walking.
Variable here and formula there;
The rhombus seemed to have no care.

The shapes had gathered to strut their stuff.
and badger those whose data was fluff!
The Quadrilateral Jam-Bor-Ee!
Was in full swing for all to see.

Rectangle first, down the aisle.
Parallel sides and a great big smile.

\[ A = bh \]

b times \( h \) is my area.
While I spin around with a tra-la-la.
2 of me makes one of him!
Triangle boasts with a great big grin.
One-half \( b \) times \( h \) is me;
\[ a \text{ fraction} \] for my fans to see!

I don’t mean to boast – I don’t mean to slam,
But here I come: Mr. Parallelogram.
I may be dull with my \( b-h \) rule;
But my slantiness makes me very cool.
Square is next, looking confident;  
With a fine felt hat, he was a snappy gent.  
With a grin that showed his happiness!  
My area is $s$ times $s$!

The crowd cheered and then subdued  
The last thing the crowd wanted to be was rude.  
What came next was the star of the show.  
Red carpet out, the lights turned low.

Trapezoid was on his way  
Three variables had caused his delay.  
\[ b_1 + b_2 \]  
In parenthesis times $h$ over two.

sum, ergo cogito
The crowd cheered and then they roared!
   *bs* and *hs* and *ss*! Good lord!
Shapes and formulas galore!
“WE WANT MORE! WE WANT MORE!”

A meek small figure then appeared.
The silence was such a pin-drop you could hear.
The crowd gaped in astonishment.
For this fellow lacked embellishment!

It’s true I lack your flare and style
You wonder why I have this smile.
You’ve played this game for oh-so-long,
Stop and ask why kids get you wrong.

The essential “area” element;
Is hidden from development.
Note my interior’s light gray-grid.
Unfortunately, this is what you’ve hid.

The implications have a consequent.
“Units betrayed” is what this has meant.
Memorization of formuli ...
I shake my head “Why oh Why!”

sum, ergo cogito
10
Tonight I come here not to preach
Not to blame but only to reach
To start anew with geometry.
To bring back students we’ve forced to flee!

Mathematical spirits I aim to lift,
But to do so requires a paradigm shift.
We’ll reach for that what our soul is yearning!
   Joy in living! Joy in learning!
The bottom base has length $b_2$, and the length of the square part is $b_1$. Therefore, the remaining length is $b_2 - b_1$.

\[ A = b_1h \]

\[ A = \frac{1}{2}h(b_2 - b_1) \]

\[
A = b_1h + \frac{1}{2}h(b_2 - b_1) \\
= b_1h + \frac{hb_2}{2} - \frac{hb_1}{2} \\
= \frac{2b_1h}{2} + \frac{hb_2}{2} - \frac{hb_1}{2} \\
= \frac{hb_1}{2} + \frac{hb_2}{2} \\
= \frac{1}{2}h(b_1 + b_2)
\]
A FURTHER THOUGHT
We can easily derive, algebraically, the formula for the area of the trapezoid, done here. But this misses the point.

It’s all about the manipulation and movement of squares and triangles. That’s the mindset. The algebra is along for the ride, but “the geometric mind” is what needs to take over here – to take precedence over the numbers.

It’s quick to realize, when you actually perform these geometric manipulations, it’s not a matter of remembering one formula or two. You don’t have to remember any!

This is part of the paradigm shift: from the algebraic to the geometric mindset.

A Closing Word
A final word on this issue, because I don’t want to leave the impression the goal of the above derivation was to arrive at the general formula for the trapezoid via geometric methods. Clearly, we can. But should we? Do we have to? Below is a worked example of finding the area of a trapezoid. Clearly, there is a place for both methods. Obviously, both methods arrive at the same answer. Wonderfully, the mental processes in doing so is radically different.
Find the Area of the Following Trapezoid

**Method #1**

\[ A = bh = (6)(4) = 24 \]

**Method #2**

\[
A = \frac{1}{2} h(b_1 + b_2) \\
= \frac{1}{2} \cdot 4(6+15) \\
= \frac{1}{2} \cdot 4(21) \\
= 42
\]
My Path

A Random Road Normally Traveled

A casual nighttime walk.
The road a narrow line.
I came upon a boulder,
That changed my state of mind.

This rock caused me to stop
And to my choice give sight.
Which direction should I choose?
To the left or to the right?

sum, ergo cogito
15
My good friend, Robert Frost,
Suggests the road not taken.
Since I was off the beaten path,
BOTH routes were forsaken!

I didn’t see it mattered.
The logic of my choice.
“Just choose it randomly!”
I said in a quiet voice.

To my left I jutted,
And to my fright I saw.
Boulders everywhere!
Seemed to be the present law!
I continued as I had, 
each move an equal chance. 
50/50 probability 
in math parlance!

I finally arrived 
A jagged, ragged, route. 
The logic of my process ...
Was sound, I had no doubt.
But then I got to thinking,
Suppose I start again.
Where would I end up?
Anywhere from One to Ten!

Just as I suspected
The flight of a different bird.
Repeat the process many times.
“Iteration” is the word!

sum, ergo cogito
To the left and right I end,
Though the center seems frequent.
What I need is tabulation.
No counting accident.

sum, ergo cogito
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The data is complete
And to my wondering eyes.
The random route I took.
Is normal in disguise!
sum, ergo cogito

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In Honor of Leap Year

2008 is a Leap Year, and today is “Leap Day”, February 29. Why do we go through the process of adjusting the calendar “every four years”? What is it we’re “putting back in alignment”?

The earth orbits the sun “once every year”. That’s our definition of “year” – one complete orbit. If we left our calendrical units here, there is no issue. However, the further use of “days” gives rise to a slight discrepancy. You see, one year doesn’t equal 365 days, but instead 365.242199 days.
But let’s round this to 365.25 days to get a better idea of what’s going on here. After one “of our years”, our calendar has moved 365 days. However, for the earth to get to it’s initial starting position, it needs an extra 1/4 of a day. It’s just short of the starting point. After the second year, it’s short 1/2 day. A third year moves the distance another 1/4 day, and finally, a fourth year leaves the earth one complete day from it’s orbital “starting point”. Leap Year intends to correct for this “drift”.

Every year, the earth falls 1/4 of a day from it’s starting point.

After four years, the earth is 1 full day from it's starting point.

The goal is for the calendar to match "the sun in the sky".

An extra day every four years is needed for "orbital restoration".
What happens if we do this? What does this look like – graphically?

**CORRECTION #1**

But let’s return to reality, now. One year does *not* equal 365.25 days. More precisely, it equals 365.242199 days. What happens if we add a day every four years? We over-compensate slightly! That is:
CORRECTION #2
So, we’ve got to somehow correct for this “over-compensation”. But how? Right now, the rule is “every 4 years, add a leap year unless the year is divisible by 100”. Let’s see what this looks like:

CORRECTION #3
Correcting for this 100-year glitch seems to put us on track – or does it? It certainly resets the bar, but does it do so accordingly? By the graph below, we seem to be losing ground now! Let’s take a guess at how the rule was modified: at 400 years, it’s clear we're at a point where, if we ignore the “100 rule”, we’re back on track.
The New Rule

“every four years is a leap year – we add a day to get us back in line. But this day over-shoots the target. Therefore, every 100 years, skip the rule. Doing this, however, erodes the ‘over-shooting’, so every 400 years, we need to add back the leap day!”

Further Playing

Let’s implement our algorithm here and see what happens. Graphing the results for the next 10,000 years yields some amazing results. The process does not stabilize! We get out of whack
around the 4,000 year! Now, likely, calendar-adjusters know this, but see no reason to make this an issue, but it is interesting to see what happens if we ourselves make it part of the rule.

What makes this most interesting to me is the official rule, regarding every 4/100/400, was made by Pope Gregory XIII in 1582 (hence the name ‘Gregorian Calendar’) long before there were calculators, satellites, and GPS systems.
The Logical Haiku of the Week
This week’s “logical haiku” is in recognition of this cosmic / calendrical calibration process!

A Revolutionary Boost

On Earth - In Sky? Peace!
Calendrical Malfunction. Quad-ennial Fix.

Quad-ennial Fix.
"Leap Year" attempts to make up for minor but accruing differences.

Our calendar marks the relation of the earth to the sun.

On Earth - In Sky? Peace!

Our "Year" does not match the Earth's orbit about the sun.

Calendrical Malfunction.
sum, ergo cogito

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Rethinking Pi-Day

Pi Day was officially March 14, but since I was on vacation, I missed the opportunity to write about it. Let’s do it now.

Pi, most people know, as 3.14 ... something. Older people may remember it as 22/7. Younger people are taught it’s something called a transcendental number, meaning it’s a number going on forever, non-repeating and non-terminating.

Contests are held, seeing who can remember the most decimal places. Why not? If it goes on forever, it’s a neat contest, right? Here are some digits after the decimal point:

1415926535 8979323846 2643383279 5028841971 6939937510 5820974944 5923078164 0628620899 8628034825 3421170679 8214808651 3282306647 0938446095 5058223172 5359408128 4811174502 8410270193 8521105559 6446229489 5493038196 4428810975 6659334461 2847564823 5786783165 2712019091 4564856692 3460348610 4543266482 1339360726 0249141273 7245870066 0631558817 4881520920 9628292540 9171536436 7892590360 0113305305 4882046652 1384146951 9415116094 3305727036 5759591953 ...

Big deal.

I’m more interested in what “pi” really means – and not memorizing someone else’s digits, but figuring this out for myself.
The concept is as easy as looking at a bike tire. If I’ve got a spoke in my tire, how big do I need to make the rim? It depends! On what? The size of the spoke. The bigger the spoke, the bigger the rim. But how much bigger? There’s the question: *how much bigger?*

If you actually measure, with a tape-measure or a string, you'll see it’s about “3 spokes”; that is, 3 spokes laid end to end about equal the size of the rim. But not exactly. It’s a bit more than three spokes. We know the relationship now as: $C = \pi d$.

How can we approximate this ourselves? Sure, we could use more precise means of measuring, and we'd get more and more precise results.

What can we do on the computer?

Suppose I draw a circle, and than add a couple items I know the answer to. I’ll draw a square about the circle, and one inside. It’s easy enough for me to calculated the areas of each square. Here, I’ve assumed the radius of my circle is one unit, so the length of
each of the outside lines is 2 units. Therefore, the area of the outside square is 4 square units. I can also find the area of the inside square as 2 units. (The specific math will show up in the next edition of =EQUALS=.)

![Diagram of a circle and a square]

But so what? What has this to do with π? I see the area of the circle is somewhere between two and four, and I also know the area of a circle is: $A=\pi r^2$ (which is why I chose my radius as ‘1’).

Fine. I know the value of π is between 2 and 4. Big deal. I already knew that.
But what happens if, instead of a square, we use a hexagon? Let’s see:

What’s going on here? The two polygons, as the number of sides grow, are getting closer to each other. But if they’re getting closer to each other, and if the circle is between them, then I should be getting a better and better approximation of \( \pi \). I am.

Let’s continue!
Clearly, my approximation becomes much better as the number of sides of the polygon increases. Having perfected the method, let’s increase the number of sides of the polygon.
I’ve always thought of \( \pi \) as 3.14159. I’ve never memorized more digits than that. What size polygon generates my idea of \( \pi \)? A 2,500-sided polygon!
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sum, ergo cogito
Logical Haiku of the Week
This method above – a modification of the “Method of Exhaustion” of Eudoxus and Archimedes from ancient times – gives rise to the “Logical Haiku” of the week ...

3.14159 ...

A Formula Known.

Infinitely thin sandwich?

Approximate π

Approximate π

I'll have a good approximation of π.

I can calculate the area of a polygon.

A Formula Known.

With a circle, I can inscribe a polygon in the circle, and the circle inside another polygon.

Infinitely thin sandwich?

sum, ergo cogito

37
Prime Numbers: A Geometric Perspective

The Absence of Geometric Rectangularization

I pushed the empty plate from in front of me at the restaurant as the waiter came to the table. “Did you enjoy your meal, sir?” “Yes – good food is much like prime numbers – there are no leftovers!” “I’d like to see that someday, sir!”

Let’s see it now.

Prime numbers are mostly thought of from an arithmetic perspective. The definition itself speaks to formulas and numbers: “a prime number is a number divisible only by itself and ‘one’.”

So 24 is not prime, because $24 = 1 \times 24, 2 \times 12, 3 \times 8,$ and $4 \times 6,$ but 29 is prime, because the only way to get to 29 multiplying integers is $1 \times 29.$

Fine.

What about ‘2’? Well, we’re told, it is prime – the only even prime – but ‘1’ is not. These are presented as though they’re self-evident definitions.
In Search of an Intuitive Understanding of Prime Numbers

Suppose there were no such things as numbers, and I ask you to break these blocks into even units.

Easy enough. There are two ways:

Fine. How about this one?

Again, easy enough. Once again, there are two ways:
Let’s try one more:

This time, there are *three* ways:

Sensing you’re tiring of the exercise, I ask one more:

*sum, ergo cogito*

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Playing around with this one for a moment, I see the only way is because does not make a “rectangle” – the blocks are not broken into even units.

Fine: let’s call the size of these blocks where the only rectangle to be made from them is the original set of blocks themselves prime – literally “of the first order (or row)”.

That’s a geometric interpretation of “prime numbers”; “of the first order”. Nary a number in sight!

But what of ‘2’? Clearly, we see it is prime, because the only way to arrange the ‘2-block’:
is this set of blocks itself.

Fine: but what about a ‘I’ block?

Above, I was talking about “blocks” (plural), and rearranging blocks (plural) in making (or not making) rectangles. We don’t have blocks here – we have a block (singular). This falls outside the realm of consideration for “prime-ness”, since we’re not dealing with blocks (plural).

A New Definition of “Prime Numbers"
with worked examples from 2 - 40
PRIME NUMBERS
The Absence of Geometrical Rectangularization

NO Geometrical Rectangularization: PRIME

Geometrical Rectangularization: NOT PRIME

sum, ergo cogito
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sum, ergo cogito
The Geometric Mind

This “geometric” interpretation / method will serve us well in dealing with binary numbers, which shall come up in the next issue of “The Geometric Mind”. In the meantime, I dedicate today’s “logical haiku” to prime numbers.

---

**The First Order**

*Numeric Array.*

*Fair Distribution Denied?*

*Hello, Prime Number!*

**Hello, Prime Number!**

Prime Numbers are NOT rectangularizable.

- Numbers can be considered geometrically, visually, or from the perspective of an array.
- Not every array can be distributed evenly among a rectangular array.

*Numeric Array.*

*Fair Distribution Denied?*
The Quadratic Stream

Swimming in the Algebraic Stream of Numbers

The Question
The current of a river is 3 miles per hour. It takes a boat a total of 3 hours to travel 12 miles upstream and return 12 miles downstream. What is the speed of the boat in still water?

In Search of an Answer
What do I know about rate, time, and distance? Rate = distance / time. I’ll start with that, and see where it leads me.

\[
\begin{align*}
r &= \frac{d}{t} \\
t &= \frac{d}{r}
\end{align*}
\]

OK … now I’ve got to translate the information I was given into this equation. How fast is my actual speed going upstream? The current provides resistance, so I’m going slower than I will be swimming downstream, where the current instead provides assistance.
rate going upstream is my rate - the rate of the stream. Going downstream, my rate of travel = my rate + the rate of the stream.

\[ t = \frac{d}{r} \]

\[ t = \frac{d}{r_{me} - r_{stream}} + \frac{d}{r_{me} + r_{stream}} \]

\[ t(r_{me} - r_{stream})(r_{me} + r_{stream}) = d(r_{me} + r_{stream}) + d(r_{me} - r_{stream}) \]

\[ t(r_{m}^2 - r_{s}^2) = dr_{m} + dr_{s} + dr_{m} - dr_{s} \]

\[ tr_{m}^2 - tr_{s}^2 = 2dr_{m} \]

\[ tr_{m}^2 - 2dx_{m} - tr_{s}^2 = 0 \]

\[ r_{m}^2 - \frac{2d}{t}r_{m} - r_{s}^2 = 0 \]

Ah – the dreaded quadratic formula. I know the general solution for this formula, so simply applying this general formula to my specific problem should lead to the solution.
Is this right? Let’s check with an example where the answer is obvious – that is, let’s suppose the stream is not flowing at all. Therefore, I’m just swimming back and forth (a total of 24 miles) in 3 hours in still water, an average speed of 8 mph. Let’s see if this formula confirms this.

\[ r_m = \frac{12}{3} \pm \frac{1}{3} \sqrt{12^2 + 0^2} = 4 \pm \frac{1}{3} \sqrt{144} = 4 \pm \frac{12}{3} = 8 \]
My General Solution:

\[ r_m = \frac{d}{t} \pm \frac{1}{t} \sqrt{\frac{d^2}{t^2} + r_s^2} \]

This leads to a pretty simple solution, but is it good to always simplify? To bring “common terms” out of the radical, for example? What happens if I leave these terms together? Let’s go back and see:

I wonder what the geometric meaning of this is – what the intuitive understanding of this is – because its elegance suggests some simple explanation. Of course, we still need to understand the “+ or -” in the solution – how is that relevant? Why do we exclude situations after the fact? Why do we say the quadratic formula yields solutions, and then we reject one of them? Is it really true if I swim at 9 mph against a 3 mph current, my effective rate is only 6 mph?

sum, ergo cogito
The Question and Answer

The current of a river is 3 miles per hour. It takes a boat a total of 3 hours to travel 12 miles upstream and return 12 miles downstream. What is the speed of the boat in still water?
\[ t = \frac{d}{r_{\text{me}} - r_{\text{stream}}} + \frac{d}{r_{\text{me}} + r_{\text{stream}}} \]

\[ t = 3, \quad d = 12, \quad r_{\text{stream}} = 3 \]

\[ 3 = \frac{12}{r - 3} + \frac{12}{r + 3} \]

\[ 3(r - 3)(r + 3) = 12(r + 3) + 12(r - 3) \]

\[ 3(r^2 - 9) = 12r + 36 + 12r - 36 \]

\[ 3r^2 - 27 = 24r \]

\[ 3r^2 - 24r - 27 = 0 \]

\[ r^2 - 8r - 9 = 0 \]

\[ (r - 9)(r + 1) = 0 \]

The "right" answer. What is this, then?

\[ r = 9 \quad r = -1 \]
Logical Haiku of the Week

This last thought gives rise to this week’s logical haiku. Going through an entire mathematical process led me to two possible solutions, and one was thrown out as irrelevant. Why? Does one ever know before the analysis the number of “viable” solutions?
sum, ergo cogito

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Challenging Intuition Directly

2,500 Years Too Late
Cleaning Up the Mess of Zeno

“THE PARADOX” PARADOX
Zeno of Elea is well known from ancient times for formulating interesting paradoxes regarding motion. Perhaps his most famous paradox is the “Tortoise and the Hare”, where he purportedly demonstrates a slow-moving tortoise, if given a head start, can never be overcome by a speedy hare.

How can this be?
Well, we’re told, surely the hare, in pursuing the tortoise, must move half the distance to the tortoise.

But in the time it takes the hare to move this distance, the tortoise itself has moved. Hence, when the hare again attempts to overtake the tortoise, it must again move halfway to the tortoise. Clearly, every time the hare moves halfway, the tortoise has moved, albeit slightly.

Hence, we’re told, the always-moving tortoise will never be overtaken by the rapidly-approaching hare, which must infinitely make up “half-distances”.

Of course, we know in reality the hare *does* overtake the tortoise, just as a fast-moving runner overtakes the plodding jogger. Why did Zeno himself not recognize his logic did not conform with reality, and wonder himself where he went wrong?

Richard Feynman, the great physicist, verbalized this wonderfully in “Surely You’re Joking, Mr. Feynman!”. While at Princeton pursuing his graduate degree, Feynman was talking with the mathematicians, who claimed you could cut up an orange into a finite number of pieces, and, putting it back together, arrive at something as big as the sun.

“Impossible”, claimed Feynman.

When given the mathematical explanation about cutting the orange, Feynman interjected: “But you said an orange! You can’t cut an orange peel any thinner than the atoms.”
When given further mathematical justification about being able to cut continuously, Feynman concluded, “No, you said an orange, so I assumed that you meant a *real* orange.”

Indeed – dealing with reality.

**A GEOMETRICAL PARADOXICAL PERSPECTIVE**

Rather than deal with this specific paradox, let’s modify the behavior of the tortoise, and say he doesn’t move at all. What of the course of action of the hare? How can we visualize it? With the ending point stable, we need only graph the halfway point between the ever-changing starting point and the stable ending point. Let’s see:

![Graph showing the halfway point between the starting and ending points.](image)

<table>
<thead>
<tr>
<th>Starting</th>
<th>Ending</th>
<th>Move</th>
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This certainly gives me a visual idea of what’s going on, but now I’d like to change the rules a bit. Rather than continuing in the same direction, always halving my distance to the goal, what would happen if I go halfway, and then wherever I am, I choose randomly to continue on in the same direction, or turn around, going in my new direction half the distance to the starting point in that direction. What would this look like? Let’s graph a few points:

![Graph showing points](image)

<table>
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<tbody>
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</table>

This new rule seems to have me going back and forth to many, many different points. What happens if I continue the pattern for a 1,000 movements? Let’s see:
As expected! I eventually hit every spot between the starting point and the ending point.

**SHIFTING TO TWO DIMENSIONS**

I’ve focused on only one direction. What happens if instead I can go in *two* dimensions? What happens if I have a square? My intuition tells me if, in one dimension I eventually landed on every point on the line, in two dimensions I should cover every point on the square.

Carrying out the procedure, I get exactly what I expected – a completely filled square:
This seems natural and intuitive: if I bounce around randomly within a certain area, eventually I will hit every point. As this was confirmed by both a straight line and a box, I suspect every shape follows suit. To be safe in confirming my theory, I decide to try the method with a triangle, and am astounded by the result:

*How can this be?*
When I Came Marching Home

A Mathematical Tribute to “Johnny”

I started marching to my home, Hurrah! Hurrah!
Zeno said “Good luck!” my friend, Hurrah! Hurrah!
“Your journey, it will be in vain,
Your head, hold up, there is no shame.”
And we’ll all feel sad, because you won’t make it home.

Ignoring math advice of all, Hurrah! Hurrah!
I started on my trip to home, Hurrah! Hurrah!
Over hill and over dale.
My progress often like a snail.
Not to be denied, I wanted to get to home.

When I came marching up the street, Hurrah! Hurrah!
The towns-folk stood with mouths agape, Hurrah! Hurrah!
“We were told it couldn’t be done.
Yet here you are, our favorite son!
And we all feel great, because you came marching home.”

That afternoon, I was in my home, Hurrah! Hurrah!
The math I sought to understand, Hurrah! Hurrah!
Half and half and half and half.
The “paradox”? I had to laugh!
I had proved Zeno wrong, because I came marching home!

sum, ergo cogito
60
The Hungarian biologist Aristid Lindenmeyer noted nature proceeds according to rules. How can one put a “grammar” to these “rules of nature” to distinguish one system from another? Lindenmeyer created such a formal-language, called the L-System, capturing relevant variables of the system.

In this example, the number of branches, the angle, and the growth reduction of each branch are captured.
2 branch: 90 degree internal angle: 60% branch scaling

sum, ergo cogito

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<table>
<thead>
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<th>2 branch: 60 degree internal angle: 60% branch scaling</th>
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</thead>
<tbody>
<tr>
<td><img src="image" alt="Tree Diagrams" /></td>
</tr>
</tbody>
</table>

*sum, ergo cogito*

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Capturing the 9th iteration of angles 20, 40, 60, 80, 100, 120, 140, 160, and 180, superimposing one over the previous.
For fun, what happens if we vary the internal angle of the tree, but maintain the growth factor:
The Algorithmic Wonder

Digital structure.
Patterns In Accord with Rules?
Similarities.

Similarities.
The ordered life has a lot in common with computer programs.

A computer program operates in accordance with rules.

Patterns In Accord with Rules?
Seashells, zebra stripes, trees, snowflakes, etc., appear as having definite patterns.

sum, ergo cogito
sum, ergo cogito
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Fall Season – Just Don’t Fall on Me!

The Advent of the Equinox

In an earlier post, I showed, with one map, three layers of causality. The position of the earth relative to the sun. The amount of sunlight during the day, and the temperature.
As today is “Fall Equinox”, I’d like to ask myself a question. What exactly is this?

I ask because I’ve never liked this definition: “The equinox is the moment when the Sun is positioned directly over the Earth’s equator, and the apparent position of the Sun at that moment.

Drawings from the internet have never helped me understand this, so let’s take matters into our own hands and take our own pictures to see what’s going on:

The next three images are to simulate the earth revolving about the sun ...

The tilt of the earth versus the incoming rays makes this winter (northern hemisphere) and summer (southern hemisphere).

The rays are perpendicular to the earth only at one place - south of the equator at the Tropic of Capricorn.

The tilt of the earth versus the incoming rays makes this winter (southern hemisphere) and summer (northern hemisphere).

The rays are perpendicular to the earth only at one place - north of the equator at the Tropic of Cancer.

sum, ergo cogito
This latter picture is the case right now, and I can see now why there is a problem with the images I’ve seen on the internet. To provide the proper perspective, the latitude lines would need to be curved, rather than horizontal as they are always portrayed.
The Tropics

At the summer solstice (northern hemisphere), the sun shines perpendicular to the earth the # of degrees as our axis of rotation.

The tropic of Cancer is the major circle of latitude 23 degrees above the equator.
There is an issue eating at me regarding all of this - something is missing. I’m certain of my logic statements above. I’m certain I
sum, ergo cogito
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understand the Tropics of Cancer and Capricorn – and, in fact, now know why the region between the two is known as “The Tropics”.

I know why we call the equinox what we do – literally, “Equal Night”. Why do we measure the solstices as we do? What is their relationship to the orbit of the earth about the sun? Am I reversing cause-and-effect somewhere in here?

For example, during the solstices, the sun strikes the earth +/- 23° from the equator. During the equinox, the sun shines directly on the equator. Therefore, between the equinox and the solstice, as the earth is flying around the sun, the sun is shining directly upon a part of the earth between 0 and 23° of the equator. It’s always shining straight down somewhere – and sometime.
Fall Season - Just Don't Fall on Me!

Brilliant green pigment.
Fading autumnal sunlight?
Color spectrum rules.

Color spectrum rules.
The leaves on the tree have turned red in the fall.

Many leaves contain a green pigment called chlorophyll.

Brilliant green pigment.

Less sunlight past the fall equinox enhances the red pigment in the leaves.

Fading autumnal sunlight?
sum, ergo cogito
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sum, ergo cogito