The Kline Identity

a newsletter of the
MORRIS KLINE SOCIETY

Volume 1
April 2007

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An Introduction

The great 20th century mathematician Morris Kline is well known for his amazing works on math history and pedagogy.

With an intense interest in mathematical history, pedagogy, and application, Morris Kline was, to me, one of the most dominant mathematical figures in the 20th century. 'Mathematical Thought From Ancient to Modern Times', 'Why Johnny Can't Add', 'Mathematics and the Physical World', and 'Mathematics and the Search for Knowledge' are but a few of the tremendous works authored by Kline. Were volume alone a criteria for greatness, he stands alone.

But more impressive than volume was the content, the focus, the drive, the joy with which each of these books shouts to the reader. Nature and the world is screaming to be understood, and it is mathematics that can - and should - lead the charge! Though he passed away in 1992, the message he left behind is an inspiring one. A clarion call? You bet!

From these thoughts came the formation of the "Morris Kline Society", with a number of initiatives in mind:

1. campaign for a "Morris Kline Commemorative Stamp";

2. re-release "*Why Johnny Can't Add*" and "*A Critique of Undergraduate Education"* (formerly known as "Why the Professor Can't Teach");

3. keep a steady stream of "Morris Kline" material in the forefront of American math education by way of the monthly newsletter "The Kline Identity";

4. create a discussion forum, discussing and debating the philosophy, application, and pedagogy of math;

5. plan a May 8, 2008 celebration, the 100th anniversary of the birth of Morris Kline, kicking off a series of events, including a campaign to make available his works on a wide basis.

In addition to the site www.morriskline.com, "The Kline Identity" newsletter will serve as means to communicate the words of the great Morris Kline.

There are no dues to be a member of the society.

Volume 2
May 2007

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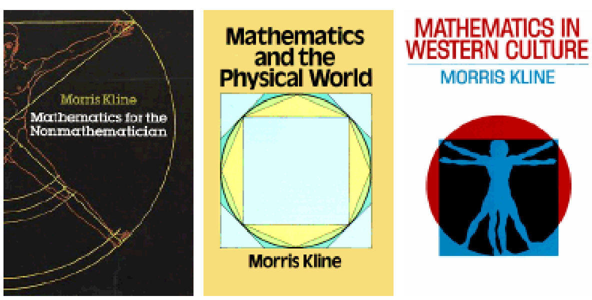
Who Was Morris Kline?

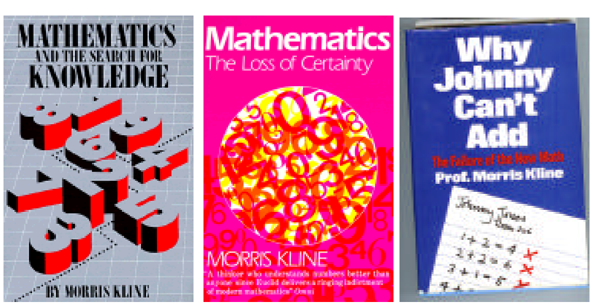
An inaugural issue demands an answer to the question, "Why this – why now – why him? Who was Morris Kline?" I will seek to answer the former by addressing the latter, letting his own words to the talking.

Who was Morris Kline? To many, Morris Kline was the author of one of the most definitive books (now a series of three books) on the history of mathematics: *Mathematical Thought from Ancient to Modern Times*:

To others, Morris Kline was the author of several books on the application of math to reality, understanding nature, and making math comfortable for many who have been traditionally labeled as "mathematically illiterate":



To others, Morris Kline was the reformist, concerned with the proper teaching of math, the status of math in curriculum, of what math means, and pedagogic considerations. According to Siobhan Roberts in "King of Infinite Space", a biography of Donald Coxeter, Morris Kline was the leading antagonist of "The New Math" revolution in the 1960s:



These are several of the books authored by Morris Kline. There were many more.

Volume and diversity of thought alone places Morris Kline in a very select classification of mathematical genius. The pedagogic considerations in how to teach math, the curriculum considerations in what to teach, and the logical considerations as to why things are the way they are, with reasonable steps to correct the mistakes of the past – to me, this is the total package.

Quoting and paraphrasing from the obituary first appearing in The New York Times, June 10, 1992:

In a 1986 editorial in Focus, a Journal of the Mathematical Association of America, he [Morris Kline] summarized some of his views: "On all levels primary, and secondary and undergraduate - mathematics is taught as an isolated subject with few, if any, ties to the real world. To students, mathematics appears to deal almost entirely with things which are of no concern at all to man".

The error, he contended, was that "mathematics is expected either to be immediately attractive to students on its own merits or to be accepted by students solely on the basis of the teacher's assurance that it will be helpful in later life." And yet, he wrote," mathematics is the key to understanding and mastering our physical, social and biological worlds."

He argued that teachers should stress useful applications of mathematics in various other fields: that they could have elementary schoolchildren deal with baseball batting averages and puzzles, get high school students work with statistics and probability, and bring college students to apply mathematics to computers and physics.

But, he said, many schoolteachers are simply unfamiliar with such teaching techniques, and the same is true of numerous college professors who were under "pressure to write research papers." He called on professional mathematics journals to print articles that instructed school and college teachers about ways of presenting such applications to their pupils and students.

"The greatest contribution mathematics has made and should continue to make was to help man understand the world about him."

Pea Soup, Tripe, and Mathematics

By Professor Morris Kline

Talk given at a meeting of the
Mathematical Association of America

November 26, 1955

Shortly after the title of this talk was announced, I received some threats on my life. I have therefore decided to change the subject of this talk from Pea Soup, Tripe and Mathematics to Tripe, Pea Soup and Mathematics. In more conventional language I wish to discuss the mechanical, meaningless mathematics that is still being taught in 90% of the colleges and universities, the mathematics that many professors would like to substitute for this established material, and finally what I believe mathematics is and therefore should be taught. My concern today is primarily with the freshman courses but many of the remarks apply to the entire undergraduate curriculum.

Let us look for a moment at the status of mathematics education. I believe that I do not have to convince anyone here that we are failing to put mathematics across. One only has to note the reactions of students to the subject, for example, their grim countenances in class, to see that we are failing. One can check this conclusion by asking his colleagues in other departments – surely an intelligent group – how they feel about the mathematics they took at school.

In trying to locate the source of the trouble, one first recognizes that students, teachers, curricula, and texts all enter the picture. The trouble may lie in any one of these quarters. Well, there can be some criticism of the students. But since about 95% of the students feel that they get nothing from their study of mathematics, the trouble can hardly be there. Criticism can be made of many teachers but since the most knowledgeable of the professors have the same trouble, this factor too can't be more than a part of the difficulty. We come therefore to curricula and texts and these may be discussed together because the texts reflect the curricula, at least these days when almost anyone can get a text published.

I proceed therefore to examine the standard college curriculum for freshmen, algebra, and trigonometry. Let me consider first whether the material we teach is knowledge in any significant sense. Does it give us any understanding of the physical world? Does it teach us anything about human physiology or social institutions? Does it help us to get along with our fellow man? Does it teach young people how to choose their mates or even the food they should eat every day? You may regard these questions as ridiculous but what I am getting at is that Horner's method, trigonometric identities, and partial fractions seem to have no bearing at all on the knowledge and problems with which even educated men should be concerned. This material has no relationship at all to human affairs in the broadest sense. In and for itself, it is meaningless material.

Of course we could be teaching something which has little or no bearing on human affairs and yet possesses intrinsic interest and beauty. The subject of ceramics might be an example. Well, I believe that I can honestly say that I like mathematics. But I hate manipulation of fractions. The laws of exponents fail to thrill me. The quadratic formula, the supposed shining light of algebra, is a huge disappointment. Partial fractions are insufferably dull. The ambiguous case is not ambiguous to me; it is decidedly disagreeable. Triangle identities keep me amused for a short time until the thought crosses my mind that I ought not to waste time and better do something significant. There is beauty in mathematics but there is no topic and no proof in college algebra or trigonometry which posses it. These subjects are dry as dust; they are mathematics at its worst.

Because students are uninformed and a captive audience, they can be gotten to absorb rubbish. But even rubbish has to be motivated. The topics we teach in college algebra and trigonometry not only are not motivated but cannot be motivated because in and for themselves they serve no purpose. We can tell students that they will use this material if they continue with mathematics. But who would want to continue with mathematics after such an introduction to it? You know the answer as well as I do. Very few. The students who reject mathematics after being exposed to algebra and trigonometry are wiser than the teachers.

I repeat that from algebra, geometry and trigonometry as such nothing follows for the students. My thought here was best expressed by Alfred North Whitehead in an address made many years ago to a group of British teachers of mathematics.

> *.... elementary mathematics ... must be purged of every element which can only be justified by reference to a more prolonged course of study. There can be nothing more destructive of true education than to spend long hours in the acquirement of ideas and methods which lead nowhere. ... [T]here is a widely-spread sense of boredom with the very idea of learning. I attribute this to the fact that they (the students) have been taught too many things merely in the air, things which have no coherence with any train of thought such as would naturally occur to anyone, however intellectual, who*

has his being in this modern world, ... the elements of mathematics should be treated as a set of fundamental ideas, the importance of which the student can immediately appreciate: that every proposition and method which cannot pass this test, however important for a more advanced study, should be ruthlessly cut out...

Over and above these objections to the standard material there is another, the topics are disconnected. From the manipulation of fractions we shift to exponents, from exponents to factoring, from factoring and perhaps the solution of equations, to complex numbers, from complex numbers to mathematical induction, from induction to permutations and combinations, from these to progressions, to binomial theorem, to partial fractions, etc. Even the trigonometry material has only an apparent coherence. What has triangle solving to do with identities, identities with trigonometric equations, the equations with trigonometric functions, the functions with the polar form of complex numbers? Surely *sin* theta and *cos* theta are involved in all of these trigonometric topics but this underlying thread is a trivial one and a technical one. From the standpoint of motivating students and large ideas there is no unity in trigonometry. How can this welter of unrelated or superficially related topics taught in college algebra and trigonometry produce more than confusion in the minds of the students.

What I have been trying to point out thus far is that the material we have been teaching to almost all of the freshman course of the past 100 years is meaningless, unmotivated, ugly and disconnected. We have taught processes which 95% of the students will never use, and even those who do go on to use it will not be effective because the material is taught mechanically. Starting with fractions in the elementary school the students learn to perform like martinets. As the students attempt to master process after process, the meaninglessness of the material frustrates them. As the years go by, the pile of meaningless operation becomes too large to bear; the entire structure collapses; and the students' progress is hopelessly blocked. Because the material has been meaningless to them, they forget readily and willingly.

I can sharpen the distinction between what we have been teaching and mathematics. The Greeks, you know, distinguished between arithmetic, the science of number, and logistics, the practice of commercial arithmetic. An analogous distinction should be made today. There is mathematics, the science and art, and there is mathematics, the craft or trade. Trade mathematics is to mathematics as plumbing is to hydrodynamics and as electrical wiring is to electromagnetic theory. We have been teaching the trade and neglecting the science and art. We have been teaching the tripe and neglecting the nourishing meat.

Unfortunately many teachers do not understand the distinction between mathematics as a trade and mathematics as an art. They themselves learned the trade as an apprentice to some journeyman and hence they know only the trade. They are the victims of a curriculum now over 100 years old and their understanding is limited to manipulative mathematics. They are entrapped by walls and tradition, educational practices, and systems and the walls are so high they can't see over them. After their indoctrination into the union of mathematical craftsmen these teachers grow old and die. Perhaps they have heard that mathematics is a gift of the Greeks and therefore fear it – in fact fear it more than their students. At any rate they never venture into mathematics proper.

The "new" books, endless repetitions of algebra and trigonometry, which appear by the dozens each year cater to these teachers and fail to challenge their narrow horizon. Ultimately, the texts become masters of the teachers and these men follow the books slavishly. Though I do not wish to-day to digress into the subject of mathematical texts, when I think of them, I am reminded of a remark which William Gilbert, the famous seventeenth century scientist makes in his De Magnete. After pointing out that Cardan in his writings described a perpetual motion machine and that many other writers did likewise, he exclaims, "May the gods damn all such shame, pilfered, distorted works, which do not but muddle the minds of the students!" May I add, and the teachers.

Up to this point, I suspect that I still have a few friends and sympathizers in the audience because fortunately, a small but interesting group of mathematics teachers has become convinced that the conventional curriculum is bad and is trying to do something about it. But I am afraid I shall lose these friends within a few minutes.

The group of people I have in mind who wish to reform the freshman curriculum have this much in their favour. They do not want to teach ideas; they do wish to exhibit and practice the reasoning processes of mathematics; and they are ready, willing, and able to break with the tradition of techniques for the sake of techniques. But I fear they have gone to the opposite extreme, for their recommendation is that we teach modern abstract mathematics. And so we find in some of the newer texts such topics as symbolic logic and Boolean algebra, postulational systems, set theory, rigorous establishment of the real numbers as Dedekind cuts, and of the complex numbers, as couples, abstract algebra, e.g., groups and fields, functions as transformations from one domain to another, and the like.

Here indeed we do find ideas and clean cut proofs. There are just a few difficulties. This material is completely over the heads of the students. It is as meaningless to them as the mechanical techniques.

The students are not prepared for such material. How can anyone understand the rigorous establishment of the real number system who can't add fractions or distinguish rational from irrational numbers? How can anyone appreciate symbolic logic who confuses all A is B with all B is A? Of what significance is Boolean algebra to students who can't square (a+b) or believe that the square root of (a^2+b^2) is equal to a+b? What can an abstract postulational system mean to students who have yet to understand deductive proof?

Advocates of the new material say that students will learn these things through the abstract approach. But the concrete, indeed, a thorough understanding of the concrete, must come before the abstract. This is the first principle of the psychology of learning. A collection of formulae such as $(p>q).(q>r)>(p>r)$ will not teach deductive reasoning. Examples of groups such as finite groups with multiplication tables will not make the concept of a real number any clearer. Students who have learned the field concept and some of its theorems will not necessarily be able to add 6 and – 7 or to add fractions. Why with a perfect knowledge of the field concept these students couldn't make change in a grocery store. They would be tied helplessly and hopelessly to a lot of, to them, meaningless assertions. Abstraction is not the first stage but the last stage of development. It may give new insight but only into concrete subject matter already learned. It may unify but it must unify what one already knows.

Whether or not the students can absorb these abstract topics I would object to them on the additional ground that abstract mathematics is empty mathematics – it is pea soup as opposed to steak. It is the form without the substance, the shell without the kernel. One does not get something for nothing even in mathematics. Certainly the concept of a field includes the rational, real and complex numbers and these distinctions are all important for any understanding of elementary mathematics.

Perhaps I can make my point clearer by suggesting that we look at the corresponding problem in geometry. Why don't we abandon triangles, circles, rectangles, parabolas, etc. and just study the concept of a geometrical figure? Hah, say the abstractionists, now you are talking my language. let's teach topology. Yes, but topology will not teach us how to obtain the area of a rectangle.

The claim has been made that the abstract approach is more efficient because several topics of conventional mathematics are encompassed in one approach. But the claim is illusory. The concrete cases must still be taught. The time that is wasted is the time spent in teaching the abstract concepts, for when the concrete cases are understood, the abstractions are readily made.

Implicit in the preceding arguments is another objection to the abstract rigorous topics listed above. This knowledge is useless knowledge, something we can't afford in today's world. The student who learns all about postulational systems will not be prepared to make the simplest use of mathematics. he will be at a loss in solving even a linear equation. We cannot afford the criticism that will be leveled at such training by the physicists, chemists, engineers, and the industrial world.

Finally, even if the students learned the abstract mathematics and its interpretations, I would still object because this material is not representative of mathematics. One indispensable element has been omitted – the physical world. Mathematics is above all an idealized formulation of physical objects and phenomena and mathematics is significant and vital because it has something to say about the physical world.

The abstractionists apparently want to keep their subject pure. They don't wish to sully it or they desire to remove the dross of the earth from which mathematics has arisen. But as they wash the ore, they keep the iron and lose the gold. No man is an island unto himself and no subject can exist in isolation. A perfect command of the English language is useless if a man has nothing to say. And pure mathematics has nothing to say. I may appropriate here Russell's famous dictum: Pure mathematics is the subject in which we never know what we are talking about nor whether what we are saying is true. And if we don't know what we are talking about the students surely won't.

Some may say that of course the physical world is there. One has to interpret the mathematics. Perhaps so. The modern abstract painters tell us that the physical world, people, and emotions are all in their works. But I defy you to find them.

Before turning to some more positive remarks I would like to make one more remark about the movement to teach rigorous abstract mathematics. This subject matter is being advertised as modern mathematics. Literally the advertisement is truthful. The concept of a field is more modern than the concept of functions. But like all advertisements, it is sales talk. It is a come-one. It is propaganda. The word modern is intended to connote something better than the old, indeed surpassing it so much that it should replace the old. But should this be done in mathematics? Real numbers, equations, functions, coordinate geometry are still not only good but vital mathematics. They have not been superseded. Fields have not replaced real numbers in science or engineering. Mathematics is not an automobile in which the 1955 chrome trimmings at least are better than the trimmings of a 1900 model; nor is mathematics a scientific theory such as a theory of light which is superseded by a new theory.

Of course emphasis and methods do change in mathematics. Much of the triangle solving usually taught in trigonometry no longer warrants the emphasis given to it and the methods of the Greek geometers by the method of coordinate geometry. But this is another matter to beginning with abstract formulations of the very concepts and methods you do want to teach. Let us not be misled by the word modern.

I have criticised, perhaps too seriously, what teachers have been doing. It is time to say what I recommend and urge. The direction in which I have been heading is no doubt already evident.

What should we teach? We want material that will provide motivation, sustain the interest of the student, exhibit the methods of the operation peculiar to mathematics, and demonstrate the chief values of mathematics. I believe that the answer is to tie mathematics closely to the study of the physical world. I do not mean that mathematics should be buried in some corner of a physical science course but rather that we should motivate interpret, and apply mathematics through fundamental physical problems and of course include wherever possible the broader implications, largely cultural, of what mathematics has accomplished. Mathematics derives from the study of nature and is valuable mainly because of what it returns to nature.

I will not undertake to provide here a full defence of this thesis but would like to give a few indications. My defence rests on history, almost always a sound guide. The beginnings of mathematics in Egypt and Babylonia were certainly physical and practical though limited in scope. The Greeks studies mathematics because it provided them with an understanding of the workings of nature. Indeed mathematics was regarded as the essence of nature and perhaps too closely identified with nature. When the study of nature was neglected during the Middle Ages, mathematics was neglected and when the study of nature was revived in the Renaissance, mathematics was revived. The seventeenth century was a period of extraordinary activity in both mathematics and science. The great lights of mathematics were also the great lights of science. And throughout the eighteenth and nineteenth centuries, the major advances in mathematics were indissolvably linked with major advances in science.. It is true that the creation of non-Euclidean geometry produced as a consequence the first sharp cleavage between mathematics and science and showed that mathematical investigations even seemingly totally unrelated to the study of the physical world may be enormously significant, but the major lesson to be derived from the history of non-Euclidean geometry is that purely mathematical and abstract studies are valuable because they may prove to be useful in the understanding and mastery of the physical world. Of course at this stage the danger also arose that mathematicians would abuse the freedom which the creation of non-Euclidean geometry granted to them and go on to create wild and valueless systems of thought. I shall return to this point shortly.

It could of course be the case that progress in mathematics was concomitant with progress in the sciences but that the mathematicians themselves worked independently. But this is not true. The great Greek mathematicians, the Ionians, the Pythagoreans, the Platonists, Eudoxus all sought to understand nature. Archimides scientific interests and inventions are famous and in fact those who are familiar with his little treatise, the Method, know that he thought physically even to discover mathematical theorems. The physical interests of Fermet, Descartes, Newton, and Leibniz are well known. May I just give the key words, optics, mechanics and astronomy. The same is true for Legendre, Lagrange and Laplace. Euler in the eighteenth century worked in almost every branch of mathematical physics, optics, hydrodynamics, analytical mechanics, lunar motions for navigating, calculus of variations, acoustic, wave motion generally. The name of Gauss immediately recalls astronomy, geodesy, electromagnetism, cartography, inventions, "Mathematics, Queen of the Sciences" is, of course, his phrase, as is the statement "Mathematics is the science of vision". Poincare and celestial mechanics are inseparable. None of these men would have appreciated or respected the distinction being made today between pure and applied mathematics.

Indeed it is fortunate that these masters did think intuitively and physically. Had the Greeks not identified geometry and physical space, they would never have developed geometry for even Euclid's presentation, the so-called rigorous version, relies upon physical facts which he failed to list as assumptions. Newton and Leibniz were fumblers as mathematicians; their logic was confused. But they had physical insight and so proceeded to make invaluable contributions. Indeed the whole history of the calculus is a comedy of errors whilst most fruitful ideas were being developed and applied. From the standpoint of rigor, Euler made the crudest mistakes but he founded the calculus of variations. That mathematics has derived its inspiration, specific problems, meaning, and goals from the physical science is no accident. I should make it clear that this appeal to history is not an argument for keeping the logic of mathematics in a confused state but it does show how much mathematical thought is dependent upon physical problems.

Of course my sketch of the forces which have produced mathematics is one-sided and incomplete. Mathematicians did create branches of mathematics which were not applied until centuries after their creation. Yes, but those men had already acquired a love for mathematics and moreover were already keenly aware of the significance of their subject. Those who created in the dark, so to speak, wasted their time as, I fear, many mathematicians are wasting time in abstract fields today. In any case we should not be teaching to freshmen, mathematics which may some day lead the way to new applications.

It is also true that many mathematical investigations though perhaps originally suggested by scientific problems extended far beyond the needs of science, e.g. Galois theory. And other investigations such as the theory of numbers have received almost no stimulus from science. These investigations are valuable, insofar as they are aesthetically attractive. But I would not try to sell the beauty of mathematics to freshmen. They must first overcome a distaste for the subject and acquire at least a modicum of comprehension.

I have thus far recommended teaching mathematics that can be related to physical problems and using the scientific aspect to give motivation and meaning. What about rigorous, deductive proof? This is a goal but not the approach. We do indeed want the students to appreciate the power of reasoning to produce new knowledge and we should continually emphasize this point whenever we have proved something significant by deduction. But there are several cautions. First we must prove something significant to the student. To prove that the multiplication of irrationals is commutative on the basis of a Dedekind cut definition of irrationals, is not significant to students who are still struggling with the very concept of the square root of 2. Nor will the proof that the order of a subgroup in a finite group divides the order of the group, mean anything to students who do not see what point there is to the very concept of a group.

Secondly, the capacity to appreciate mathematical rigor is a function of the age of the student and is independent of the age of mathematics. Some of the recent books on calculus which have stressed rigor (Begle and Menger) are the heights of absurdity for this reason.

Thirdly, rigorous proof is the polish on mathematics. It is the last stage of a development. The very concept of deductive proof in itself is the product of only one civilization of hundreds, the civilization of the Greeks. This fact in itself shows that rigorous proof is a sophisticated, difficult, and even esoteric concept. Hence it is certainly not easy for the students. Moreover, the rigorous proof comes after the discovery and full intuitive understanding of the theorem to be proved. This is true historically of mathematics in the large and is true of the very process of creating individual theorems. Today we are trying to emphasize the gilt and the polish and we are leaving out the substance. We are throwing away the heritage of mathematics.

To illustrate how these general recommendations may be put into practice let me give just one example of how one might treat a subject commonly taught in elementary mathematics and which must be taught if the students is ever to do anything at all with mathematics, namely, coordinate geometry. This subject is treated about as shabbily as any I know of. The tendency today is either to shorten the material to the bare mathematical essentials of the old technical presentations or to give some highly sophisticated concept of function and then apply it to curves.

What is the major idea we wish to put across? It is that we can write an equation for a curve which not only fully represents the curve but that we can then study the properties of curves systematically and very effectively with the full powers of algebra and analysis.

To motivate the interest in curves one must give a few examples of the occurrences of these curves in genuine physical problems. e.g. the conic sections in reflectors, lenses, planetary motion, and projectile motions. Let's pose genuine problems e.g. to design the cable of a bridge, when will the moon eclipse the earth (simplified)?, where will a projectile land? Let's not just suppose students appreciate the need to solve those problems but discuss them at the very beginning.

Then let us pose the problem of how we can derive facts about these curves. We know their geometric definitions but these do not seem to provide any answers. It should be clear to the students before proceeding that we need method. I would then discuss the Euclidean methods and show how clumsy and specialized they are. Let us illustrate this for the circle, e.g. we might prove of the circle involving chords and tangents. I would then digress from the mathematics proper to present Descartes' interest in method from mathematics, science, and philosophy. This discussion could even enter into some problems of philosophy, then extend perhaps dependant upon the interests of the group. With a liberal arts group the subject should be pursued further than with a group of mathematics majors.

We could then teach coordinates and equations of curves in the usual way and do some geometrical problems by the methods of coordinate geometry, preferably the same problems previously used to illustrate the clumsiness of Euclidean geometry. Incidentally we should teach the algebra we need as we need it and thereby take care of some of the topics in the usual college algebra course.

As we teach the various topics let's do several things. First let us make genuine applications of each idea. Secondly, the teacher and text should include some physical explanations which will make the applications intelligible and real to the student. Thirdly, the applications should not be a series of disconnected problems. They must be organized to make a coherent body of problems. A few classes of applications thoroughly treated will do more than a thousand unrelated ones.

For example, some coordinate geometry texts do mention the focusing property of a parabola. But nothing is said about light rays and so the students feel uneasy about the entire application. One should certainly point out that rays are an approximate method for treating light and radio waves and point out that the law of reflection for rays is a physical law which very fortunately fits in with the purely geometric property of the parabola, namely, that the radius vector to a point on the parabola and a line parallel to the axis and through that point make equal angles with the normal. Many applications of curve and equation to light should be described. If these physical and mathematical ideas are fully discussed the students will enjoy the application, see the role of mathematics, and willingly undertake further problems on the same idea.

As another example, when teaching parametric equations we should teach projectile motion. Again the surrounding physical facts must be taught. The possibility of using parametric equations for physical applications rests on physical fact that the horizontal and vertical motions of a projectile are independent of each other. Also, the mathematics should be used to predict facts about the motion, e.g. range, maximum height. At this point the power of mathematical reasoning can readily be stressed.

After the students have gotten some part of the usual coordinate geometry material behind them, it is time to boost their interest by introducing some new ideas. The possibility of investigating higher dimensions and even a brief start on this subject will pique them and keep them curious and anxious for more mathematics.

Somewhere in the course one should point out that new curves can be discovered by means of equations. This is now done in routine problems which presuppose the curve exists. Let's make this topic an exploration into new territory. It is new for the students. Here we have the opportunity to do something exciting.

And somewhere near the end of the course one must point out and stress the most important value of coordinate geometry for modern times. Coordinate geometry is the basis of applied mathematics. The students have already seen how reflector shapes, paths of projectiles and planets have been represented by equations and then studied. But coordinate geometry makes is possible to take any path and shape and study it mathematically. Let's talk about the hulls of ships, the fusilages and wings of aeroplanes, the paths of radio waves in space, the shapes of bridge cable and roadways and point out that we can now study all of these physical shapes by efficient mathematical methods. Let's refer to the calculus yet to come as a powerful new tool which works on the equations of these curves and surfaces.

The treatment I have recommended for coordinate geometry should be applied to every undergraduate mathematical subject right through differential equations. I might remark incidentally that I envisage wiping out college algebra as a subject in and for itself.

May I summarize the point of view toward mathematics which I believe is proper for elementary instruction. Math is an abstract formulation of ideas suggested by the physical world; it is the artful use of reasoning processes to infer and deduce new facts about the physical world; it is a series of significant assertions about the physical world; and it contains implications about almost all the arts and sciences. All four aspects of mathematics must be taught. Techniques are boring but necessary details which must be properly subordinated to the ideas, the reasoning and the conclusions. Moreover they should be taught when needed for some larger goal.

I know that as I suggest treatments of the various branches of elementary mathematics which will bring out these values of mathematics many of you are mentally protesting, where do we get all this material. Well, I have given some samples of the scientific relationships and cultural influences in my book Mathematics in Western Culture. With the little work I did along these lines I could have written a book twice the size. Unfortunately my commitments do not permit me to devote much time to gathering and organizing such material but I am certain that it exists in abundance. It must be collected and put into the curricula and textbooks of the future.

Here then is the direction in which we must go to improve the mathematics curriculum and here is the road for textbook writers to pursue. Let me echo Gilbert again, 'May the gods damn all the sham, pilfered distorted works which have but muddled the minds of the students' and here I add, may the gods bless those undertakings which seek to enrich the teaching of mathematics.

From the Quote Department
Quotes from Books and Interviews

The Stem AND the Flower

“By neglecting motivation and application the pedagogues have caused mathematics education to suffer. These men have presented the stem but not the flower and so have failed to present the true worth of what they are teaching. They call upon students to fight battles but do not tell them why they are engaged in them. Even the United States Army knows better.”

Why Johnny Can’t Add

Analogies in Context

“Of course, as we have already seen, reasoning by analogy may lead to trouble. Bulls and cows look alike but anyone who tries to pet a bull, thinking it is a cow, will soon discover the essential danger in analogy.”

Mathematics and the Physical World

Tragedies

“There are tragedies caused by war, famine, and pestilence. But there are also intellectual tragedies caused by limitations of the human mind. This book relates the calamities that have befallen man’s most effective and unparalleled accomplishment, his most persistent and profound effort to utilize human reason – mathematics.”

Mathematics: The Loss of Certainty

A Return to the Fountainhead

“The emphasis on research in the last thirty or forty years has diverted manpower from teaching and so has cut off the flow into the fountain of all our educational efforts, the teaching in the colleges. Perhaps rather belatedly we shall develop sincere and capable cultivators of mathematics - a science and an art - who will recognize that exposition is as vital in their medium as it is in painting, music, and literature.”

A Critique of Undergraduate Mathematics

Be Careful What You Ask For

“The back-to-basics movement, in part a reaction to the new math, is not the solution to decent mathematics education. It means to me the meaningless drill in techniques that was common twenty and more years ago. That type of education failed, as is evidence by the attitude of most intelligent, educated people towards mathematics. It will almost surely fail again. It may not be worse than the new math but it is surely not better.”

Quoted in "Interview of Morris Kline" by G.L. Alexanderson, from Mathematical People edited by D.J. Albers and G.L. Alexanderson.