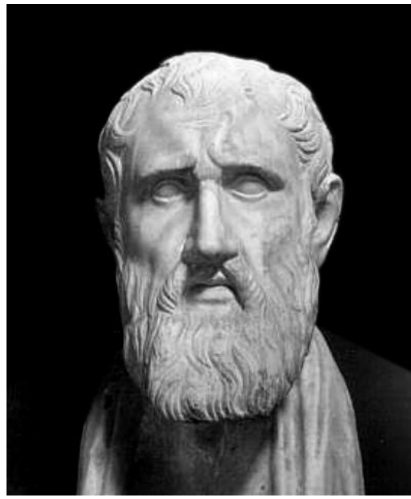
2,500 YEARS TOO LATE

CLEANING UP THE MESS OF ZENO

Good morning. I've very happy to be here today to talk about a topic dear to my heart – *fractals*. But rather than start with "fractals", I want to tell you a story about a name famous in the annals of mathematical paradoxes: Zeno. You all know the name.



"THE PARADOX" PARADOX

Zeno of Elea is well known from ancient times for formulating interesting paradoxes regarding motion. Perhaps his most famous paradox is the "Tortoise and the Hare", where he purportedly demonstrates a slow-moving tortoise, if given a head start, can never be overcome by a speedy hare.

How can this be?

Well, we're told, surely the hare, in pursuing the tortoise, must move half the distance to the tortoise.

But in the time it takes the hare to move this distance, the tortoise itself has moved. Hence, when the hare again attempts to overtake the tortoise, it must again move halfway to the tortoise. Clearly, every time the hare moves halfway, the tortoise has moved, albeit slightly.

Hence, we're told, the always-moving tortoise will never be overtaken by the rapidly-approaching hare, which must infinitely make up "half-distances".

Of course, we know in reality this is ridiculous. We know in reality the hare *does* overtake the tortoise, just as a fast-moving runner overtakes the plodding jogger. Why did Zeno himself not recognize his logic did not conform with reality, and wonder himself where *he* went wrong?

Richard Feynman, the great physicist, verbalized this wonderfully in "*Surely You're Joking, Mr. Feynman!*". While at Princeton pursing his graduate degree, Feynman was talking with the mathematicians, who claimed you could cut up an orange into a finite number of pieces, and, putting it back together, arrive at something as big as the sun.

"Impossible", claimed Feynman.

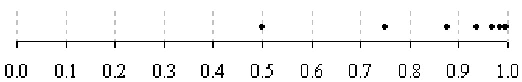
When given the mathematical explanation about cutting the orange, Feynman interjected: "But you said an orange! You can't cut an orange peel any thinner than the atoms."

When given further mathematical justification about being able to cut continuously, Feynman concluded, "No, you said an orange, so I assumed that you meant *a real orange*."

Indeed – dealing with reality.

A GEOMETICAL PARADOXICAL PERSPECTIVE

Rather than deal with this specific paradox, let's modify the behavior of the tortoise, and say he doesn't move at all. What of the course of action of the hare? How can we visualize it? With the ending point stable, we need only graph the halfway point between the ever-changing starting point and the stable ending point. Let's see:

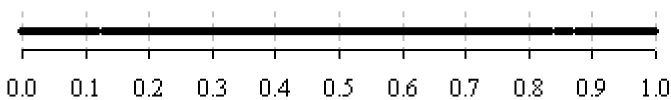


Starting	Ending	Move
0.00000	1.00000	0.50000
0.50000	1.00000	0.75000
0.75000	1.00000	0.87500
0.87500	1.00000	0.93750
0.93750	1.00000	0.96875
0.96875	1.00000	0.98438
0.98438	1.00000	0.99219
0.99219	1.00000	0.99609
0.99609	1.00000	0.99805
0.99805	1.00000	0.99902
0.99902	1.00000	0.99951

This gives me a visual idea of what's going on, but now I'd like to change the rules a bit. Rather than continuing in the same direction, always halving my distance to the goal, what would happen if I go halfway, and then wherever I am, I choose *randomly:* to continue on in the same direction, or turn around, going in my new direction half the distance to the starting point in that direction. What would this look like? Let's graph a few points to get an idea:

Starting	Ending	Move
0.00000	1.00000	0.50000
0.50000	0.00000	0.25000
0.25000	1.00000	0.62500
0.62500	0.00000	0.31250
0.31250	1.00000	0.65625
0.65625	0.00000	0.32813
0.32813	0.00000	0.16406
0.16406	0.00000	0.08203
0.08203	1.00000	0.54102
0.54102	0.00000	0.27051
0.27051	1.00000	0.63525

This new rule seems to have me going back and forth to many, many different points. What happens if I continue the pattern for a 1,000 movements? Let's see:

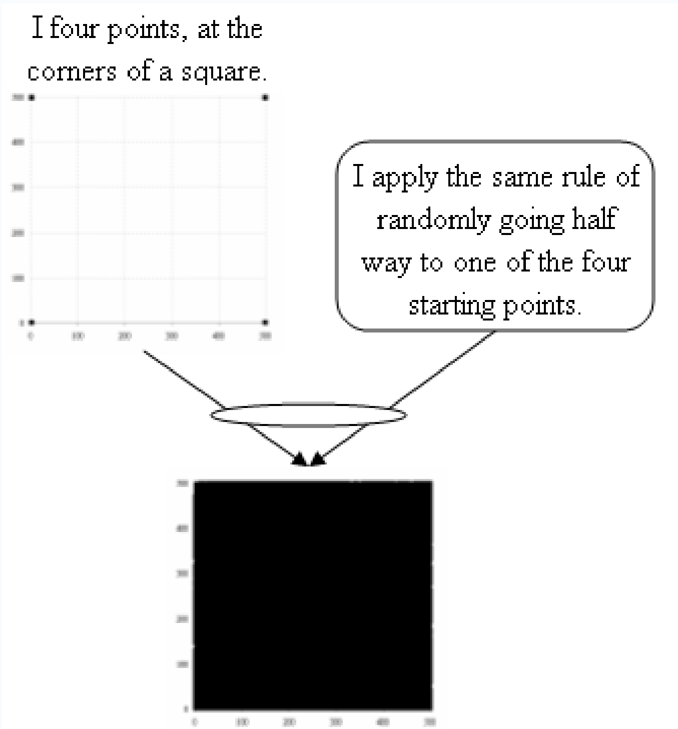


As expected! I eventually hit every spot between the starting point and the ending point.

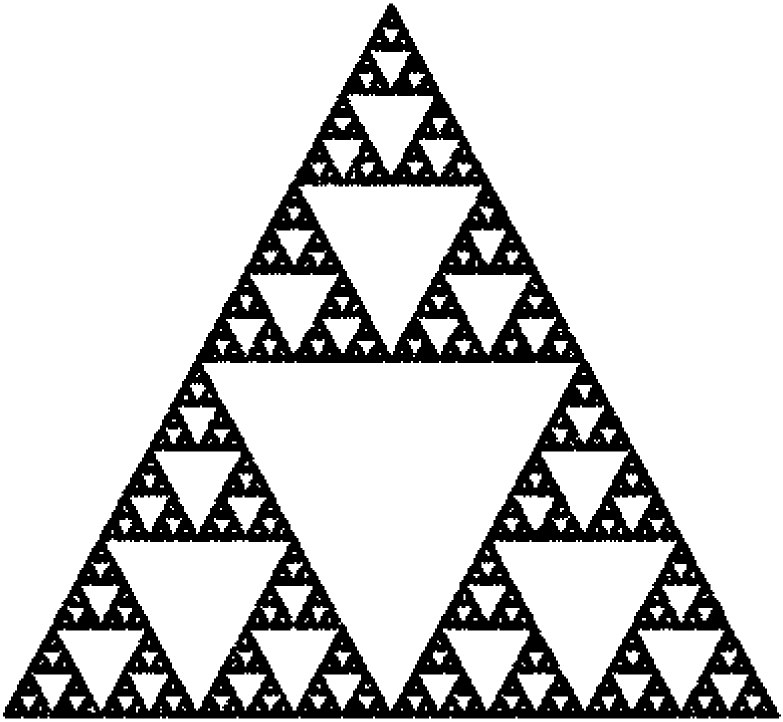
SHIFTING TO TWO DIMENSIONS

I've focused on one dimension. What happens if instead I can go in *two* dimensions? What happens if I have a square? My intuition tells me if, in one dimension I eventually landed on every point on the line, in two dimensions I should cover every point in the square.

Carrying out the procedure, I get exactly what I expected – a completely filled square:



This seems natural and intuitive: if I bounce around randomly within a certain area, eventually I will hit every point. As this was confirmed by both a straight line and a box, I suspect every shape follows suit. To be safe in confirming my theory, I decide to try the method with a triangle, and am astounded by the result:

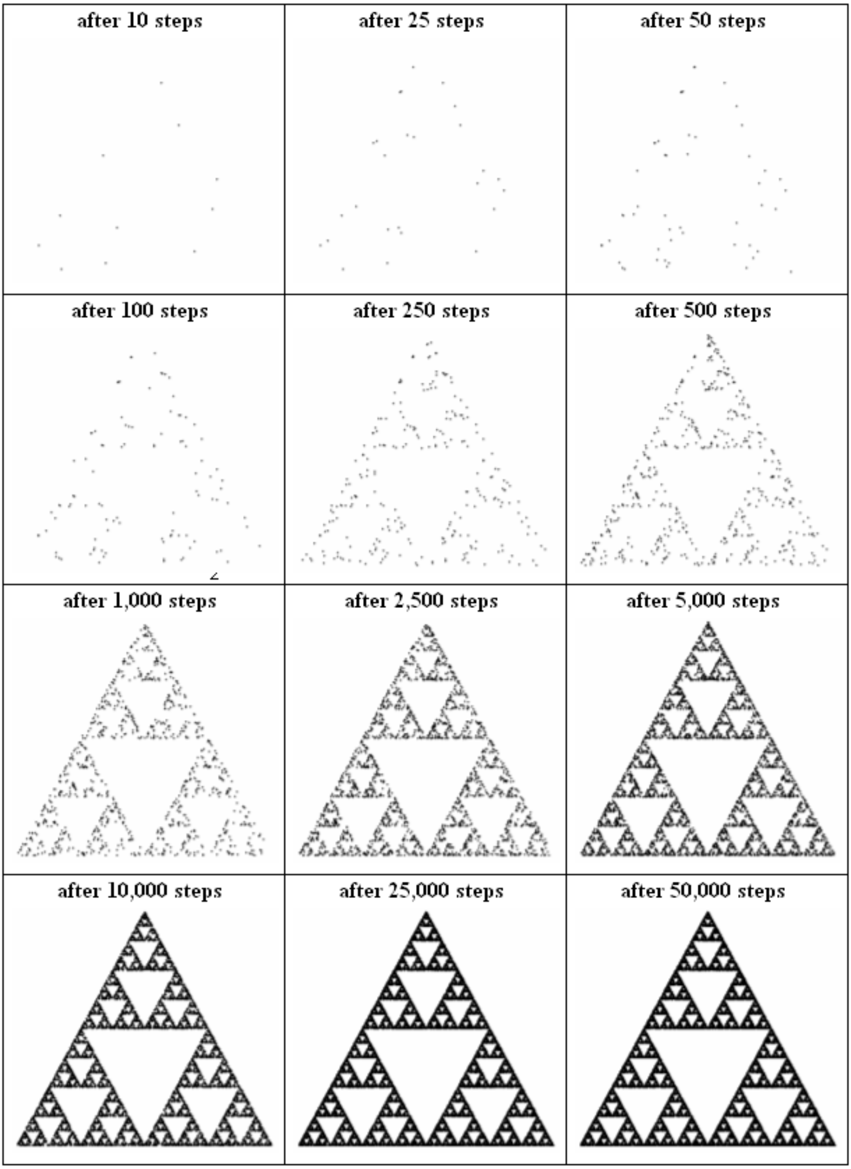


How can this be?

This makes no sense, particularly given the solid straight line and the filled square earlier. But this was the result of moving 50,000 times. Let's "slow it down", and capture the results to see how this took place:

RANDOM STEPPING IN TWO DIMENSIONS

From 10 to 50,000 Steps

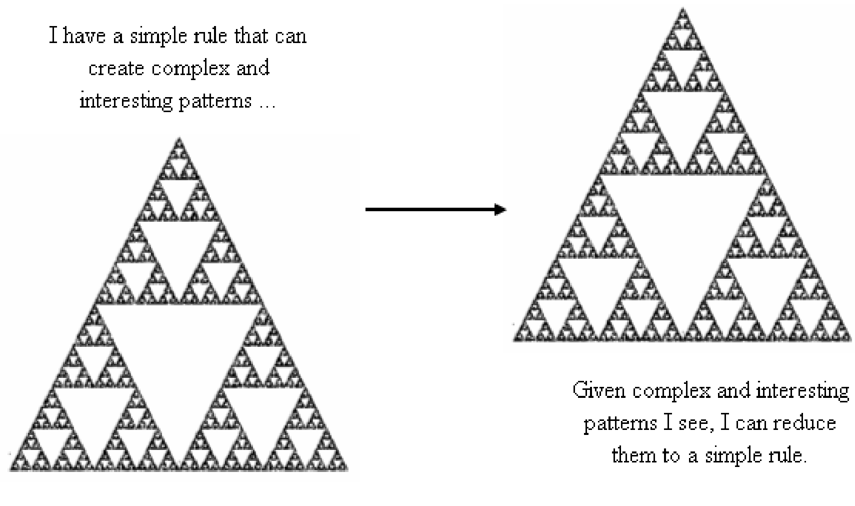


THE IDEA OF FRACTAL COMPRESSION

The result of this experiement was this: I had a very simple rule, and after applying that rule, I arrived at complex behavior that was similar at every step. All that was added was more steps. If I had to carry the image in a suitcase on a plane, I wouldn't need the image; all I would need is the process.

The thought that arose from this interesting – and non-intuitive – experiment was this: if, by using a simple rule I can generate complex and interesting patterns, then *can the reverse hold true?*

That is: can I reduce behavior I see in reality to a simple set of rules – "fractal compression", if you will. In other words:



For example, suppose I had an image of a fern. The image of the fern is composed of millions of bits of information. Additionally, as I zoom in on the image, I lose the clarity of the fern.

However, if I could capture the nature – the essence – of the fern in a simple rule, I not only save exponentially in the amount of data needed to be saved to represent the fern, but I additionally do not lose clarity when I zoom in on the fern, because additional magnification includes the rule itself!